FLUID MECHANICS SOLUTION

SEM 4 (CBCGS-MAY 2019)

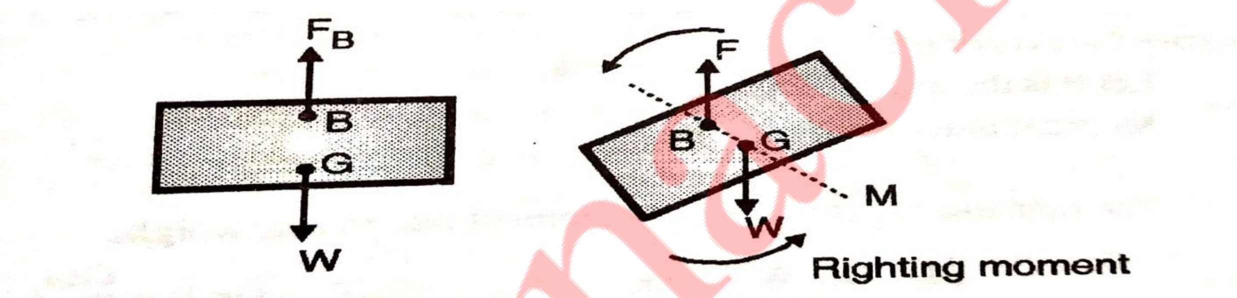
BRANCH-MECHANICAL ENGINEERING

Q 1) a) Explain conditions of equilibrium of floating bodies. **(05)**

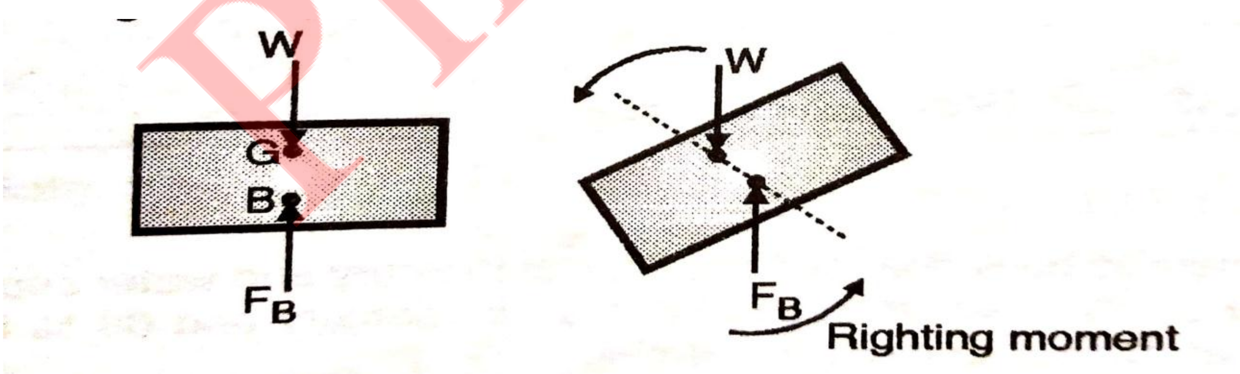
Solution:

Conditions of stable equilibrium:

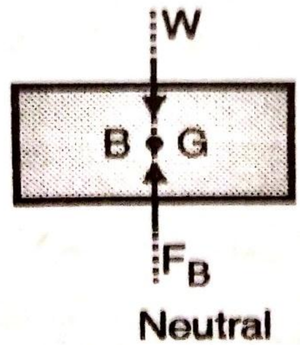
1) Stable equilibrium:
When centre of buoyancy is lies above the centre of gravity, submerged body is stable.



2) Unstable equilibrium:
When centre of buoyancy is lies below centre of gravity, submerged body is in unstable equilibrium.



3)Neutral equilibrium:



When centre of buoyancy and centre of gravity is coincide, body is in neutral equilibrium.

Q 1) b) Explain i) velocity function ii) Stream function. **(05)**

Solution:

Velocity Potential :

The velocity potential is defined as a scalar function of space and time such that its negative derivative with respect to any direction gives the fluid velocity in that direction. It is denoted by ϕ(phi). Thus mathematically the velocity potential is defined as:

$$\varphi = f(x, y, z, t) \text{ ...for unsteady flow,}$$

$$\text{and,} \varphi = f(x, y, z) \text{ ...for steady flow;}$$

Such that:

$$u = -\frac{\partial \varphi}{\partial x}$$

$$v = -\frac{\partial \varphi}{\partial y}$$

$$w = -\frac{\partial \varphi}{\partial z}$$

where, u, v and w are the components of velocity in the x, y and z directions respectively.

Stream Function :

The stream function is defined as a function of space and time, such that its partial derivative with respect to any direction gives the velocity component at right angles to this direction. It is denoted by ψ (psi).

In case of two-dimensional flow, the stream function may be defined mathematically as :

$$\psi = f(x, y, t) \text{ ...for unsteady flow, and}$$

$$\psi = f(x, y) \text{... for steady flow,}$$

such that: $$u = \frac{\partial \psi}{\partial x}$$

$$u = -\frac{\partial \psi}{\partial x}$$

Q 1) c) Do the following velocity components Represent physically possible flow? (05)

$$\mathbf{u = x^2y\,; \quad v = 2zy - xy^2\,; \quad w = x^2 - z^2y}$$

Solution:

$$u = x^2y\,; \quad v = 2zy - xy^2\,; \quad w = x^2 - z^2y$$

Differentiating

$$\frac{\partial u}{\partial x} = 2xy \qquad \frac{\partial v}{\partial y} = 2z - 2xy \quad \frac{\partial w}{\partial z} = -2zy$$

Using continuity equation,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$2xy + 2z - 2xy - 2zy = 0$$

$$2z - 2zy \neq 0$$

Since continuity equation is not satisfied, flow is not possible.

Q 1) d) An aircraft is flying with a velocity of 200 m/s through the still air at -15°C. Find the stagnation pressure, if the mass density of the air is 1.08 $\mathrm{kg/m^3}$.Take pressure of the air as 80 kPa. Take R $= 287\ \mathrm{J/kgK}$. (05)

Solution:

Velocity of air, $V = 200\ \mathrm{m/s}$
Pressure, $p_1 = 80\ \mathrm{kPa} = 80 \times 10^3\ \mathrm{N/\,m^2}$

Temperature $t_1 = -15°\mathrm{C} = -15 + 273 = 258\ \mathrm{K}$

R= 287 J/kgK , and assuming γ =1.4

Velocity of sound C $= \sqrt{\gamma RT} = \sqrt{1.4 \times 287 \times 258}$

$$C_1 = 321.97 \text{ m/s}$$

Mach number

$$\mathrm{M}_1 = \frac{V_1}{C_1} = \frac{200}{321.97} = 0.621$$

Stagnation pressure

$$\mathrm{P_s} = \mathrm{P_1}\left(1 + \left(\frac{\gamma-1}{2} M_1^2\right)\right)^{\frac{\gamma}{\gamma-1}}$$

$$= 80\left(1 + \left(\frac{1.4-1}{2} \times 0.621^2\right)\right)^{\frac{1.4}{1.4-1}}$$

$$= 103.75 \text{ kPa}$$

Q 1) e) Explain surface tension and capillarity. **(05)**

Solution:

Surface tension:

Surface tension is caused by the force of cohesion at the free surface. A liquid molecule in the interior of the liquid mass is surrounded by other molecules all around and is in equilibrium. At the free surface of the liquid, there are no liquid molecules above the surface to balance the force of the molecules below it. Consequently, as shown in Fig. 1.18, there is a net inward force on the molecule. The force is normal to the liquid surface. At the free surface a thin layer of molecules is formed. This is because of this film that a thin small needle can float on the free surface (the layer acts as a membrane).

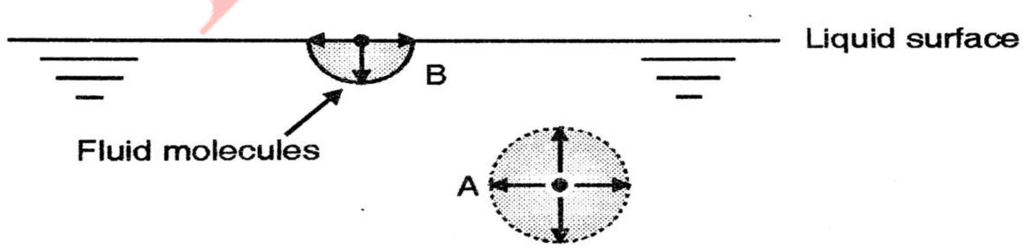


FIG: SURFACE TENSION

Capillarity :

Capillarity is a phenomenon by which a liquid (depending upon its specific gravity) rises into a

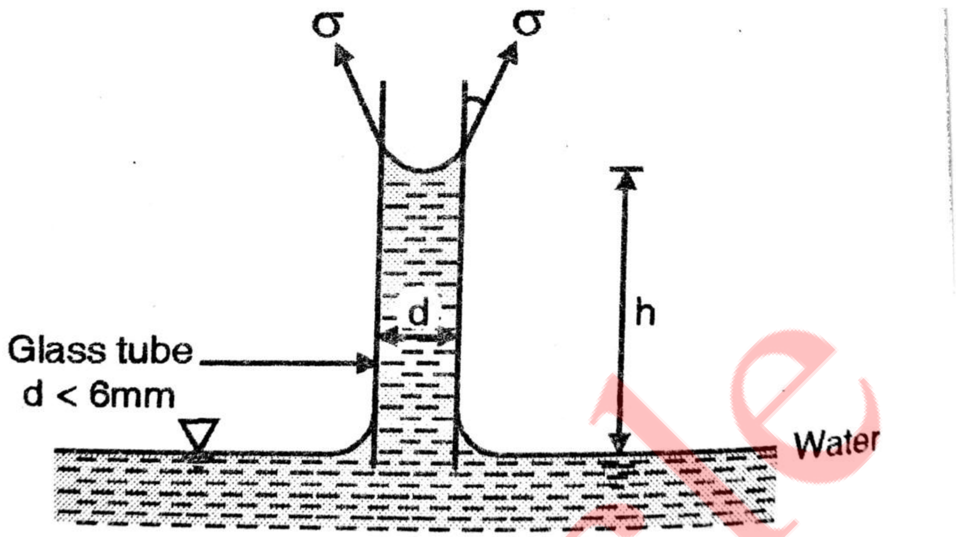


FIG: CAPILLARY RISE

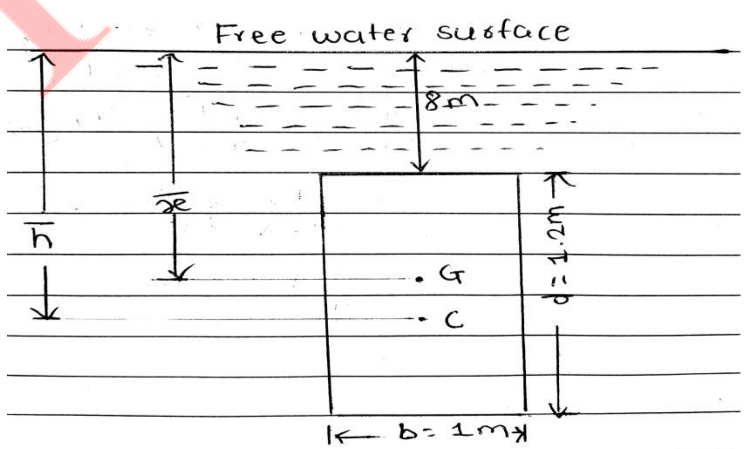
thin glass tube above or below its general level. This phenomenon is due to the combined effect of cohesion and adhesion of liquid particles.

Q 2) a) A heavy car plunges into a lake during an accident and lands at the bottom of the lake on its wheels. The door is 1.2 m high and 1 m wide, the top edge of the door is 8 m below the free surface of the water. Determine the hydrostatic force acting on the door approximating it as a vertical rectangular plate and the location of centre of pressure considering bottom of the lake surface as horizontal. **(10)**

Solution:

Width of the plane surface, b = 1 m

Depth of the plane surface, d = 1.2 m



Area of the plane surface,

$$A = b \times d = 1 \times 1.2 = 1.2\, m^2$$

$$\bar{x} = 8 + \frac{1.2}{2} = 8.6 \text{ m}$$

(i) Total pressure P:

Using the relation:

$$\text{P} = \text{wA}\bar{x} = 9.81 \times 1.2 \times 8.6$$

$$= 101.23 \text{ KN}$$

(ii) Centre of pressure, $\bar{\text{h}}$:

Using the relation:

$$\bar{\text{h}} = \frac{\text{I}_\text{G}}{\text{A}\bar{\text{x}}} + \bar{x}$$

$$\text{I}_\text{G} = \frac{\text{bd}^3}{12} = \frac{1\times1.2^3}{12} = 0.144\text{m}^4$$

$$\bar{\text{h}} = \frac{0.144}{1.2\times8.6} + 8.6 = 8.614 \text{ m}$$

$$\bar{\text{h}} = 8.614 \text{ m}$$

Q 2) b) What is Venturimeter ? Derive expression of the discharge through venturimeter. (10)
Solution:

Venturimeter :

A venturimeter is one of the most important practical applications of Bernoulli's theorem. It is an instrument used to measure the rate of discharge in a pipeline and is often fixed permanently at different sections of the pipeline to know the discharges there.

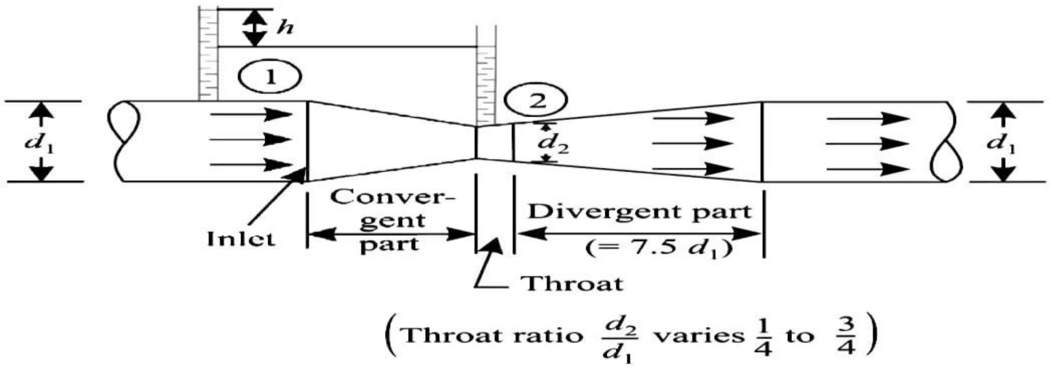


FIG: VENTURIMETER

Expression for rate of flow:

Fig shows a venturimeter fitted in horizontal pipe through which a fluid is flowing.

Let, $D_1 =$ Diameter at inlet or at section 1,

$A_1 =$ Area at inlet $(= \frac{\pi}{4} d_1^2)$

$P_1 =$ Pressure at section 1,

$V_1 =$ Velocity of fluid at section 1,

and D2, A2, p2, and V2 are the corresponding values at section 2.

Applying Bernoulli's equation at sections 1 and 2, we get:

$$\frac{P_1}{w} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{w} + \frac{V_2^2}{2g} + Z_2 \qquad \text{...(i)}$$

Here, $z_1 = z_2$ ... since the pipe is horizontal.

$$\frac{P_1}{w} + \frac{V_1^2}{2g} = \frac{P_2}{w} + \frac{V_2^2}{2g}$$

$$\frac{P_1 - P_2}{w} = \frac{V_2^2}{2g} - \frac{V_1^2}{2g} \qquad \text{...(ii)}$$

But, $\frac{P_1 - P_2}{w}$ = Difference of pressure heads at sections 1 and 2 and is equal to h.

$$\frac{P_1 - P_2}{w} = h$$

Substituting this value of $\frac{P_1 - P_2}{w}$ in eqn. (ii), we get:

$$h = \frac{V_2^2}{2g} - \frac{V_1^2}{2g} \qquad \text{...(iii)}$$

Applying continuity equation at sections 1 and 2, we have:

$$A_1V_1 = A_2V_2 \quad \text{or} \quad V_1 = \frac{A_2V_2}{A_1}$$

Substituting the value of V_1 in eqn. (iii), we get:

$$h = \frac{V_2^2}{2g} - \frac{\left(\frac{A_2V_2}{A_1}\right)^2}{2g} = \frac{V_2^2}{2g}\left(1 - \frac{A_2^2}{A_1^2}\right)$$

$$h = \frac{V_2^2}{2g} - \left(\frac{A_1^2 - A_2^2}{A_1^2}\right) \text{ or } V_2^2 = 2gh - \left(\frac{A_1^2}{A_1^2 - A_2^2}\right)$$

$$V_2 = \sqrt{2gh\left(\frac{A_1^2}{A_1^2 - A_2^2}\right)} = \left(\frac{A_1}{\sqrt{A_1^2 - A_2^2}}\right)\sqrt{2gh}$$

Discharge $\quad Q = A_2V_2 = A_2\left(\frac{A_1}{\sqrt{A_1^2 - A_2^2}}\right) \times \sqrt{2gh}$

$$Q = \left(\frac{A_1A_2}{\sqrt{A_1^2 - A_2^2}}\right) \times \sqrt{2gh} \qquad \text{...(A)}$$

Or $\quad Q = C\sqrt{h}$

where, C = constant of venturimeter

$$= \left(\frac{A_1A_2}{\sqrt{A_1^2 - A_2^2}}\right) \times \sqrt{2g}$$

Eqn. (A) gives the discharge under ideal conditions and is called theoretical discharge. Actual discharger (Q_{act}) which is less than the theoretical discharge (Q_{th}) is given by:

$$Q_{act} = C_d \times \left(\frac{A_1A_2}{\sqrt{A_1^2 - A_2^2}}\right) \times \sqrt{2gh}$$

where, Cd = Co-efficient of venturimeter (or co-efficient of discharge) and its value is less than unity (varies between 0.96 and 0.98)

Q 3) a) A 45° reducing bend is connected in a pipeline, the diameter at the inlet and outlet of the bend being 400 mm and 200 mm respectively. Find the force exerted by water on the bend if the intensity of pressure at inlet is 215.8 KN/m^2. The rate of flow of water is 500 lit/sec. (10)

Solution:

Discharge through bend Q $= 500\ \mathrm{lit/sec} = 0.5\ \mathrm{m^3/s}$

$$A_1 = \frac{\pi}{4}(0.4)^2 = 0.1257\ \mathrm{m}^2$$

$$A_2 = \frac{\pi}{4}(0.2)^2 = 0.0314\ \mathrm{m}^2$$

Pressure at inlet $\rho_1 = 215.8\ \mathrm{kN/\ m^2}$

Velocity at inlet and outlet

$$V_1 = \frac{Q}{A_1} = \frac{0.5}{0.1257} = 3.98\ \mathrm{m/s}$$

$$V_2 = \frac{Q}{A_2} = \frac{0.5}{0.0314} = 15.92\ \mathrm{m/s}$$

Using bernoulli's equation

$$\frac{P_1}{\Upsilon} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\Upsilon} + \frac{V_2^2}{2g} + Z_2$$

$$\frac{215.8\times10^3}{1000\times9.81} + \frac{3.98^2}{2\times9.81} + Z_1 = \frac{P_2}{1000\times9.81} + \frac{15.92^2}{2\times9.81} + Z_2$$

Where, $Z_1 = Z_2$

$$21.99 + 0.807 = \frac{P_2}{1000\times9.81} + 11.91$$

$$P_2 = 10.887 \times 9810 = 106.8 \times 10^3\ kN/\ \mathrm{m}^2$$

By momentum principle

Net force in x-direction = mass flow rate × change in velocity in x-direction

$$P_1A_1 - P_2A_2\cos\theta + F_x = \rho Q(V_2\cos\theta - V_1)$$

$$215.8 \times 10^3 \times 0.1257 - 106.8 \times 10^3 \times 0.0314\cos45° + F_x = 1000 \times 0.5(15.92\cos45° - 3.98)$$

$$24754.76 + F_x = 3638.57$$

$$F_x = -21116.19\ \mathrm{N} = -21.11\mathrm{kN}$$

$$= 21.11\mathrm{kN}\ (\leftarrow)$$

Q 3) b) A fluid of viscosity 8 poise and specific gravity 1.2 is flowing through a circular pipe of diameter 100 mm. The maximum shear stress at the pipe wall is 210 N/m2. Find: (i) The pressure gradient, (ii) The average velocity, and (iii) Reynolds number of flow. (10)

Solution:

Viscosity of fluid, μ = 8 poise = 0.8 Ns/m^2

Specific gravity = 1.2

∴ Mass density, ρ = 1.2 × 1000 = 1200 kg/m^2

Diameter of the pipe, D = 100 mm = 0.1 m

Maximum shear stress, τ_0 = 210 N/m^2

(i) The pressure gradient, $\frac{\partial p}{\partial x}$:

We know, $$\tau_0 = -\frac{\partial p}{\partial x}.\frac{R}{2}$$

Or $$\tau_0 = -\frac{\partial p}{\partial x}.\frac{(0.1/2)}{2}$$

∴ $$\frac{\partial p}{\partial x} = -\frac{210\times4}{0.1} = -8400\ \ \mathrm{N/m^2}\ \text{per m}$$

(ii) The average velocity, $\bar{u}$:

We know, $$\bar{u} = \frac{1}{2}\,u_{max}$$

$$= \frac{1}{2}\left[-\frac{1}{4\mu}.\frac{\partial p}{\partial x}.R^2\right]$$

$$= \frac{1}{2}\left[-\frac{1}{4\times0.8}\times(-8400)\times(0.1/2)^2\right]$$

$$= 3.28\ \mathrm{m/s}$$

(iii) Reynolds number, R_e :

$$R_e = \frac{\rho VD}{\mu} = \frac{1200\times3.28\times0.1}{0.8} = 492$$

Q 4) a) Two reservoirs with a difference in elevation of 15 m are connected by two pipes in series. The pipes are 150 m long of 20 cm diameter and 200 m long of 25 cm diameter respectively. The friction factors for the two pipes are respectively 0.020 and 0.019. Determine discharge through pipe considering both major and minor losses. (10)

Solution:

Given:

$d_1 = 0.2\text{ m}$ $\quad L_1 = 150\text{ m}$ $\quad f_1 = 0.020$

$d_2 = 0.25\text{ m}$ $\quad L_2 = 200\text{ m}$ $\quad f_2 = 0.019$

Area of pipe

$$A_1 = \frac{\pi}{4}(d_1)^2 = \frac{\pi}{4}(0.2)^2 = 0.0314\text{ m}^2$$

$$A_2 = \frac{\pi}{4}(d_2)^2 = \frac{\pi}{4}(0.25)^2 = 0.049\text{ m}^2$$

Using continuity equation

$$A_1V_1 = A_2V_2$$

$$V_1 = \left(\frac{A_2}{A_1}\right)V_2$$

$$= \left(\frac{0.049}{0.0314}\right)V_2 = 1.56V_2$$

$$V_2 = 0.641V_1$$

Total head H = Major loss + minor loss

$$15 = h_1 + h_2$$

Major loss $h_1 = \left(\frac{flv^2}{2gd}\right)_1 + \left(\frac{flv^2}{2gd}\right)_2$

$$= \frac{0.020\times150\times(V_1)^2}{2\times9.81\times0.2} + \frac{0.019\times200\,(0.641V_1)^2}{2\times9.81\times0.25}$$

$$= 0.764\,V_1^2 + 0.318\,V_1^2$$

$$h_1 = 1.082\,V_1^2$$

Minor loss $h_2 = \text{Loss at entry} + \text{Loss due to expansion} + \text{Loss at exit}$

$$= 0.5\frac{V_1^2}{2g} + \frac{(V_1 - V_2)^2}{2g} + \frac{V_2^2}{2g}$$

$$= 0.5\frac{V_1^2}{2g} + \frac{(V_1 - 0.641V_1)^2}{2g} + \frac{0.641_1^2}{2g}$$

$$= 1.5\frac{V_1^2}{2g}$$

$$h_2 = 0.75\, V_1^2$$

$$H = h_1 + h_2$$

$$15 = 1.082\, V_1^2 + 0.75\, V_1^2$$

$$V_1^2 = \frac{1.832}{15} = 0.122$$

$$V_1 = 0.349 \text{ m/s}$$

Discharge $\quad Q = A_1 V_1 = 0.0314 \times 0.349$

$$= 0.0109 \text{ m}^3/s$$

Q 4) b) What do you mean by boundary layer separation? What is the effect of pressure gradient on boundary layer separation? (07)

Solution:

(i) When a solid body is immersed in a flowing fluid, a thin layer of fluid called boundary layer is formed, adjacent to the solid body. In this thin layer of fluid, the velocity varies from zero to free stream velocity in the direction normal to the solid body.

(ii) Along the length of the solid body, the thickness of the boundary layer increases. The fluid layer adjacent to the solid surface has to do work against surface friction at the expense of kinetic energy.

(iii) This loss of kinetic energy is recovered from the immediate fluid layer in contact with the layer adjacent to the solid surface through momentum exchange process. Thus the velocity of the layer goes on decreasing.

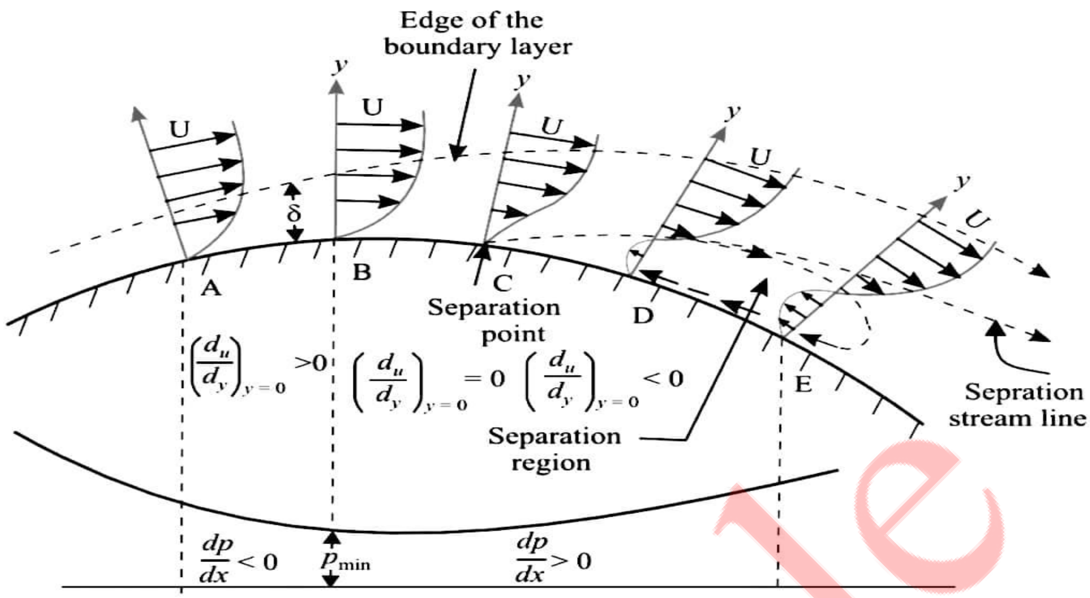


FIG: BOUNDARY LAYER SEPERATION

(iv) Along the length of solid body, at a certain point a stage may come when the boundary layer may not be able to keep sticking to the solid body, if it cannot provide kinetic energy to overcome the resistance offered by the solid body. In other words, the boundary layer will be separated from the surface. This phenomenon is called the boundary layer separation.

Effect of pressure gradient on boundary layer separation:

(i) As the fluid flows round the surface (the area of flow decreases) it is accelerated over the left hand section until at point B the velocity just outside the boundary is maximum and the pressure is minimum (as shown by the graph below the surface). Thus from A and B the pressure gradient is negative. As long as $\frac{dp}{dx} < 0$, the entire boundary layer moves forward.

(ii) Beyond B (i.e. along the region BCDE), the area of flow increases and hence velocity of flow decreases; due to decrease of velocity the pressure increases (in the direction of flow) and hence the pressure gradient $\frac{dp}{dx}$ is positive i.e. $\frac{dp}{dx} < 0$. The value of the velocity gradient $\frac{du}{dy}$ at the boundary is zero at the point C, this point is known as a separation point.

Q 4) c) Define mach number and give its significance. **(03)**

Solution:

Mach Number:

The Mach number is an important parameter in dealing with the flow of compressible fluids, when elastic forces become important and predominant. Mach number is defined as the square root of the ratio of the inertia force of a fluid to the elastic force.

1. Subsonic flow: Mach number is less than 1.0 (or M < 1); in this case V < C.

2. Sonic flow: Mach number is equal to 1.0 (or M = 1); in this case V = C.

3. Supersonic flow: Mach number is greater than 1.0 (or M > 1); in this case V > C

When the Mach number in flow region slightly less to slightly greater than 1.0, the flow is termed as transonic flow.

Q 5) a) Describe compressible flow through a convergent-divergent nozzle. (05)

Solution:

Let, $P_2(= P_c) =$ Pressure in the throat when the flow is sonic for given pressure P_1

(i) When the pressure in the receiver, $P_3 = P_1$, there will be no flow through the nozzle, this is shown by line a in Fig.

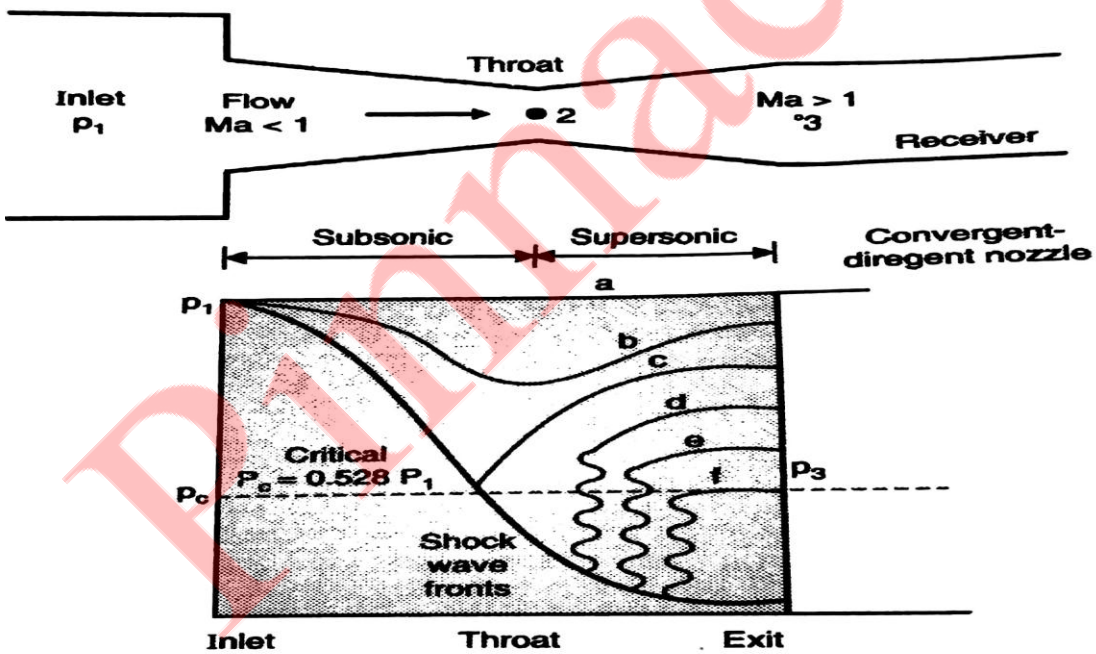


FIG : CONVERGENT-DIVERGENT NOZZLE

(ii) When the receiver pressure is reduced, flow will occur through the nozzle. As long as the value of P_3 is such that throat pressure P_2 is greater than the critical pressure 0.528 P_1, the flow in the converging and diverging sections will be subsonic. This condition is shown by line 'b'.

(ii) With further reduction in P_3, a stage is reached when P_2 is equal to critical pressure pc = 0.528 P_1, at this line M = 1 in the throat. This condition is shown by line ‘c’. Flow is subsonic on the upstream as well the downstream of the throat. The flow is also isentropic.

(iii) If P_3 is further reduced, it does not effect the flow in convergent section. The flow in throat is sonic, downstream it is supersonic. Somewhere in the diverging section a shock wave occurs and flow changes to subsonic (curve d). The flow across the shock is non-isentropic. Downstream of the shock wave the flow is subsonic and decelerates.

(iv) If the value of P_3 is further reduced, the shock wave forms somewhat downstream (curve e).

(v) For P_3 equal to pj, the shock wave will occur just at the exit of divergent section.

(vi) If the value of P_3 lies before pf and pj oblique waves are formed at the exit :

Q 5) b) What do you understand by displacement and momentum thickness? Determine displacement thickness and momentum thickness for the following velocity distribution.

$$\frac{u}{U_0} = \frac{3}{2}\left(\frac{y}{\delta}\right)$$ **(08)**

(i) The displacement thickness can be defined as follows:

“It is the distance, measured perpendicular to the boundary, by which the main/free stream is displaced on account of formation of boundary layer.”

Or

“It is an additional “wall thickness” that would have to be added to compensate for the reduction in flow rate on account of boundary layer formation”.

(ii) Momentum thickness is defined as the distance through which the total loss of momentum per second be equal to if it were passing a stationary plate. It is denoted by θ.

It may also be defined as the distance, measured perpendicular to the boundary of the solid body, by which the boundary should be displaced to compensate for reduction in momentum of the flowing fluid on account of boundary layer formation.

$$\frac{u}{U_0} = \frac{3}{2}\left(\frac{y}{\delta}\right)$$

Displacement thickness

$$\delta^* = \int_0^\delta \left[1 - \frac{u}{U_0}\right] dy$$

$$= \int_0^{\delta} \left[1 - \frac{3y}{2\delta}\right] dy$$

$$= \left[y - \frac{3y^2}{4\delta}\right]_0^{\delta}$$

$$= \delta - \frac{3}{4}\delta$$

$$= \frac{1}{4}$$

momentum thickness,

$$\theta = \int_0^{\delta} \frac{u}{U_0}\left(1 - \frac{u}{U_0}\right) dy$$

$$= \int_0^{\delta} \frac{3y}{2\delta}\left(1 - \frac{3y}{2\delta}\right) dy$$

$$= \int_0^{\delta} \left[\frac{3y}{2\delta} - \frac{9y^2}{4\delta^2}\right] dy$$

$$= \left[\frac{3y}{2\delta} - \frac{9y^2}{4\delta^2}\right]_0^{\delta}$$

$$= \frac{3\delta}{4} - \frac{3\delta}{4}$$

$$= 0$$

Q 5) c) A flow field is characterized by $\Psi = x^3 y$. Determine the velocity potential function ϕ for the flow if the flow is irrotational. **(05)**

Solution:

Stream function $\Psi = x^3 y$

$$\frac{\partial \phi}{\partial y} = -v = x^3 \qquad \text{...(i)}$$

$$\frac{\partial \phi}{\partial x} = -u = 3x^2 y \qquad \text{...(ii)}$$

Now integrating Equation (i),

$\phi = \frac{x^4}{4} + f(x)$ ...(iii)

Differentiating w.r.t. x,

$\frac{\partial \phi}{\partial x} = \frac{x^3}{3} + f'(x)$ From equation (ii)

$\frac{\partial \phi}{\partial x} = 3x^2 y$

$3x^2 y = f'(x)$

Integrating $-3\frac{y^2}{2} + C = f(x)$

Put value of $f(x) = -3\frac{y^2}{2}$ in equation (iii),

$\phi = \frac{x^4}{4} - 3\frac{y^2}{2} + C$

Q 6) a) An aeroplane is flying at a height of 20 km, where the temperature is -40℃. The speed of the plane corresponding to $\mathbf{M = 1.8}$ Assuming $\mathbf{k = 1.4}$ and $\mathbf{R = 287 J/kgK}$. Find the speed of the plane. (05)

Solution:

Given: M=1.8 k=1.4 R=287J/kgK.

$$T = -40 + 273 = 233°K$$

We know velocity of sound,

$$C = \sqrt{\Upsilon RT}$$

$$= \sqrt{1.4 \times 287 \times 233} = 305.97 \text{ m/s}$$

But $M = \frac{V}{C}$ or $1.8 = \frac{V}{305.97}$

$$V = 550.74 \text{ m/s}$$

Q 6) b) Explain streamlined body and bluff body. (05)

Solution:

(i) Streamlined body:

A body whose surface coincides with the stream lines when placed in a flow, is called a streamlined body (Fig. a). In this case flow separation takes place only at the trailling edge or rearmost part of the body. The wake formation zone behind a streamlined body is very small, as a consequence of which the pressure drag will be very small. In such a body although due to greater surface of the body the skin friction increases but the net effect is a significant reduction of total drag.

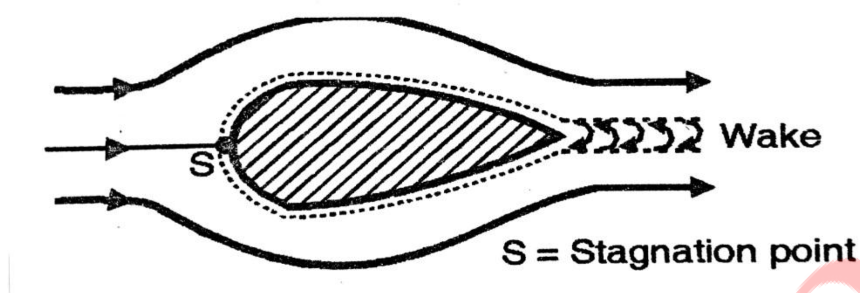


FIG: STREAMLINE BODY

A body may be streamlined at low velocites but may not be so at higher velocities, also when placed in a particular position in flow but may not be so when placed in another position. Streamlined shapes are used for the wings of aeroplanes and for the blades of marine propellers and rotary axial flow machines.

(ii) Bluff body:

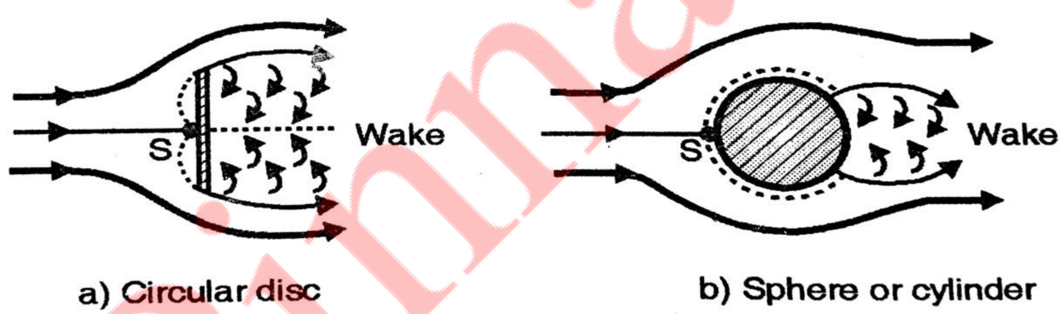


FIG: BLUFF BODY

A body whose surface does not coincide with streamlines when placed in a flow, is called a bluff body (Fig. b). In this case there is extensive boundary layer separation accompanied by a wake with large scale eddies. Due to large wake formation, the resulting pressure drag is very large as compared to the drag due to friction on the body.

Q 6) c) State and prove Bernoulli's theorem for streamline flow. (10)

Solution:

Bernoulli's equation states as follows:

"In an ideal incompressible fluid when the flow is steady and continuous, the sum of pressure energy, kinetic energy and potential (or datum) energy is constant along a stream line."

Mathematically,`

$$\frac{p}{W}+\frac{V^2}{2g}+Z=\text{constant}$$

where,

$\frac{p}{W}$ = Pressure energy,

$\frac{V^2}{2g}$ = Kinetic energy, and

Z = Datum (or elevation) energy

Proof:

$$\frac{dp}{\rho}+VDV+g=0$$

This is the required Euler's equation for motion, and is in the form of differential equation.

Integrating the above eqn, we get :

$\frac{dp}{\rho}\int dp+\int V.dV+\int g.dz=\text{constant}$

$\frac{P}{\rho}+\frac{V^2}{2}+gZ=\text{constant}$

Dividing by g, we get:

$\frac{P}{\rho g}+\frac{V^2}{2g}+Z=\text{constant}$

$\frac{P}{w}+\frac{V^2}{2g}+Z=\text{constant}$

In other words,

$$\frac{P_1}{w}+\frac{V_1^2}{2g}+Z_1=\frac{P_2}{w}+\frac{V_2^2}{2g}+Z_2$$

Which proves Bernoulli's theorem